Andrew McNaughton

MANUAL

OF THE

LABORATORY

OF

MATHEMATICS & DYNAMICS

Engineering Building

McGILL UNIVERSITY

BY

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CONTENTS.

MATHEMATICAL LABORATORY.

1. THE BALANCE.

EXPERIMENT.—To use the balance in determining the mass of a given object.

Apparatus.—A balance. Box of weights. Camel's hair brush. A brass weight marked ... grains.

Method I.—Level the case if necessary (probably it is sufficiently level). Dust the pans with the brush. Bring the pans to rest, turn the key gently and set the beam swinging. The pointer would move backwards and forwards along the scale for a long time; do not wait for it to come to rest in order to determine the position of rest; find this by observing the extreme positions of the pointer on the scale. Read the scale from left to right, counting each whole division as 10. Take an even number of consecutive readings on one side of the scale and an odd number on the other (say 2 and 3, or 3 and 4); find the mean of each set and then the mean of the two results; this will give the position of rest of the pointer. It is best that the pointer should move over about half (the middle half) of the scale.

Arrest the beam when the pointer is near the middle of the scale in order to avoid jarring the beam. The beam must always be at rest when any change is made in the body to be weighed, or in the weights, or in the position of the rider.

Again, free the beam and repeat your work. Take as the position of rest the mean of the several results.

Enter the results thus :—

EXPERIMENT No. 1. I.

First trial.

Readings on left. Readings on right.

Mean . . . Mean . . .

Mean of both . . .

Second trial.

Readings on left Readings on right.

Mean . . . Mean . . .

Mean of both . . .

Mean of the two trials . . .

II.—To find the sensitiveness.

Place a rider at the division 1 on the beam. The mass of a rider is 1 centigramme. Hence, putting the rider at the division 1 is equivalent to putting one milligramme (.001 gramme) in the pan. Set the beam oscillating and again find the position of rest. The difference between this result and that of I. gives the sensitiveness of the balance for 1 milligramme when the pans are empty. At least two or three determinations should be made.

The sensitiveness will probably not be quite the same when the pans are loaded, but the result you have now obtained will be of use in checking your subsequent weighings.

Enter results thus :—

EXPERIMENT No. 1. II.

Enter readings as in I.
Position of rest is now . . .
" " " was . . .
∴ sensitiveness for 1 milligramme = . . .

III.—To weigh the given body (the . . . grain weight) and hence to calculate the number of grains in a gramme.

Place the body in one of the pans (say the left) and the gramme weights in the other. Always move the weights with the forceps, never with the fingers. After putting a weight in the pan turn the key until the pointer *just begins* to move. The direction will show whether the weight is too large or too small. Continue to adjust the weights until you are within a few milligrammes of the true result. The final weighing must be made with the rider, the case being closed. Determine the position of rest with the rider in one position, then shift the rider along the beam, and again find the position of rest. One of these two positions should be on one side and the other on the other side of the position of rest when the pans are empty. From these results find the number of milligrammes to be added to the weights in the pan.

Enter results thus :—

EXPERIMENT No. 1. III.

Position of rest with rider at . . . is . . .
" " " " . . . is . . .
" " pans empty is . . .

∴ number of milligrammes to add to weights in pan = . . .
Weights in pan . . .
Total weight . . .
13.43206
Hence 1 gramme = $\frac{\dots}{\dots}$ = . . . grains.

IV.—To weigh the given object (a piece of wood marked 1), the result to be corrected for the buoyancy of the air.

Find the apparent weight as in III. Each side is buoyed up by the weight of the air displaced by the bodies in the pans. Let the apparent weight be W grammes. Then the weight of the given body is very nearly W grammes. Hence its volume is $\frac{W}{s}$ c. c. nearly, s being the number of grammes in a c. c. of the wood. (This may be assumed as .56). Hence for the air displaced by the body the correction is $+(.0012)\frac{W}{s}$, .0012 grammes being assumed as the weight of 1 c. c. of air. Similarly the correction for the air displaced by the brass weights is $-(.0012)\frac{W}{s_1}$ where s_1 is the number of grammes in 1 c. c. of the brass, which may be taken as 8.4.

Enter results thus :—

EXPERIMENT NO. 1. IV.

Apparent weight . . .
Correction for air displaced by the body, + . . .
" " " " weights − . . .
Total correction . . .
Corrected weight . . .

(Results of parts I, II, III, IV of this Experiment to be returned together. Each student to sign his name.)

2. SPECIFIC GRAVITIES.

Experiment.—To find the specific gravity (I) of a body which sinks in water, (II) of a body which floats in water, (III) of a liquid.

Apparatus.—An ordinary balance arranged to weigh a body when immersed in water. Distilled water. Silk thread. Copper or other wire.

Method I.—Place the body (brass weight marked 2) in the balance pan, and weigh it in air. Call this weight W_1. Attach the body, by means of a light string or wire. to the hook above the pan, and weigh it again. Let the weight now be W_2. Keeping the body still attached to the hook, bring underneath the vessel of distilled water, having previously exhausted the air from the water. See that no air bubbles remain attached to the body when thus immersed in water. It may be found that the beam will not oscillate freely when the body is in the water; if so, adjust the weights so as to bring the pointer to the position of rest when the pans are empty, without depending upon the method of oscillations. Let the weight in the water be W_3. Then $W_2 - W_3$ is the weight of the displaced water. Hence the specific gravity of the body is

$$\frac{W_1}{W_2 - W_3}.$$

All weighings must be corrected for displacement of air whenever such correction is appreciable.

The W_2 differs from the W_1 by the weight of the string or wire. W_3 must be corrected for the weight of water displaced by the string or wire. The specific gravity of silk is about 1, that of iron about 7¾, that of copper about 9.

The body and the water should be of the same temperature as the surrounding air. Let this be t° C. The above value of the specific gravity is that of the body when referred to water at the temperature t°. To reduce it to what it would be if referred to water at 4° C. we must multiply by the specific gravity of water at t°. Let this be called ρ. Then the specific gravity of the body at the temperature t° is

$$\frac{W_1}{W_2 - W_3} \times \rho.$$

The water should be freed from air by boiling, or by means of an air pump.

Find the specific gravity of the given brass weight, marked 2, by two determinations, (*a*) hanging it up by a silk cord, and (*b*) by a copper wire.

Enter results as follows :—

EXPERIMENT No. 2. I.

(*a*) With silk cord.

Observed.	Corrections if any.	Corrected values.
W_1		
W_2		
W_3		

$t = \ldots{}^\circ$ $\rho = \ldots$

Specific gravity $= \dfrac{\ldots}{\ldots - \ldots} \times \ldots = \ldots$

(Enter *b* in same way.)

II.—When the body is lighter than water it must be pulled down into the water by means of a sinker. Weigh the body (marked 3) first in the scale pan (W_1). Attach it and the sinker (2) to the balance hook, the sinker being below the given body. Weigh the two *with the sinker in water* (W_2). Add more water so as to cover the given body and weigh again (W_3). Then the specific gravity of the body at the observed temperature when referred to water at 4° is

$$\frac{W_1}{W_2 - W_3} \times \rho$$

where ρ is the specific gravity of water at the temperature of the experiment.

Find the specific gravity of the given body using silk thread.

Enter results as under I.

III.—To find the specific gravity of a liquid, take a heavy body ("sinker") which will sink in the given liquid as well as in water. Attach the sinker to the hook of the balance and weigh it in air (W_1). Next weigh it in water (W_2). Dry the sinker and weigh it in the given liquid (W_3). Then $W_1 - W_3$ and $W_1 - W_2$ are the weights of equal volumes of the given liquid and water. Hence, the specific gravity of the given liquid at the temperature of the experiment, referred to water at 4° is

$$\frac{W_1 - W_3}{W_1 - W_2} \times \rho$$

where ρ is the specific gravity of water at the temperature of the experiment.

Find the specific gravity of the given liquid, using silk thread to suspend the sinker.

Enter results as under I.

(Each student to sign his name.)

3. THE SPECIFIC GRAVITY BOTTLE.

EXPERIMENT.—To find, using a specific gravity bottle, the specific gravity (I) of a liquid, (II) of a solid in fragments.

Apparatus.—A balance. A specific gravity bottle. Distilled water.

Method I.—To find the specific gravity of a liquid.

Weigh the bottle (W_1). The specific gravity of the glass may be assumed to be 3. Hence, find the correction to the weighings for the air displaced by the glass. (It may not be appreciable.) Fill the bottle with water; insert the stopper, wipe the outside dry and weigh (W_2). Empty the water, fill with the given liquid and weigh (W_3). Then the sp. gr. s is

$$\frac{W_3 - W_1}{W_2 - W_1}.$$

The given liquid and the water are assumed to be of the same temperature, and the above fraction represents the specific gravity referred to water at the temperature of the experiment. To reduce to water at 4° C. multiply by ρ the specific gravity of the water at the temperature of the experiment.

Enter results thus :—

EXPERIMENT No. 3. I.

$W_1 = \ldots$, corrected value $= \ldots$

$W_2 = \ldots$, " " $= \ldots$

$W_3 = \ldots$, " " $= \ldots$

Temperature = ...

ρ for this temperature = ...

$$\therefore s = \frac{W_3 - W_1}{W_2 - W_1}\ \rho = \ldots$$

II.—To find the specific gravity of small fragments of a solid.

Weigh the fragments (W_1). Fill the bottle with water, wipe dry and weigh (W_2). Put the fragments in the bottle, fill up with water if necessary, wipe dry and weigh (W_3).

Then

$$s = \frac{W_1}{W_1 + W_2 - W_3} \cdot \rho$$

Enter results thus :—

EXPERIMENT No. 3. II.

$W_1 = \ldots$, corrected value $= \ldots$

$W'_2 = \ldots$

$W_3 = \ldots$

$W_2 - W_3$ corrected for air displaced by difference of brass weights = ...

Temperature = ...

ρ for this temperature = ...

$\therefore s = \ldots$

(Each student to sign his name.)

4. JOLLY'S SPRING BALANCE.

Experiment.—To find (I) the weight of a small body, (II) the specific gravity of a small body, (III) the specific gravity of a liquid.

Apparatus.—A scale of millimetres is etched on a versical strip of mirror glass in front of which hangs a long spiral spring carrying two pans. A white bead is strung on the wire which connects the upper pan to the spring; the top of the bead corresponds to the pointer of an ordinary balance, and in reading its position on the scale the line of sight must pass over the top of the bead and the top of its image in the mirror. In this way any error of parallax in reading the scale is avoided. The small platform, adjustable as to height, is intended to carry a vessel of distilled water, or of other liquid, in specific gravity determinations. The spring is intended to support weights up to 6 grammes, but a more delicate one carrying up to 3 grammes, and a less delicate one carrying up to 10 grammes, are provided, as is also a small glass sinker, which may be suspended in the place of the two pans.

Method I.—To find the weight of a given small object. Place it in the upper pan. Give a little support to the pan at first and let it down gently. Read the scale when the spring comes to rest, or deduce the position of equilibrium from the oscillations along the scale. Now remove the body and in its place put weights from the box until the bead comes to the same position as at first.

Enter results thus :—

Experiment 4. I.

Scale reading in millimetres when body is in pan . . .
Weight of body in grains . . .

II.—Within certain limits the extension of the spring is nearly proportional to the force which produces it. To verify this, read the scale when the pans are empty. Next place in the upper pan any small weight from the box, say 10 grains, and again read the scale. Add another weight and take the new reading.

Enter results thus :—

EXPERIMENT NO. 4. II.

First scale reading	...			
Second " "	...	for weight	...	grains.
Third " "	...	" "	...	"
Ratio of weights	...			
" " extensions caused by weights ...				

With this spring each millimetre of extension corresponds to ... grain.

III.—To find the specific gravity of a given solid. In this experiment the lower pan must be immersed in distilled water throughout. Adjust the levelling screws so that the pan will hang quite freely in the water, and adjust the platform so as to bring the pan within about a quarter or half an inch from the bottom of the water. The pans being now empty, read the scale. Place the given solid in the upper pan, move the platform until the lower pan is at about the same depth in the water as before, and read the scale. Move the body to the lower pan, again adjust the platform and read the scale. Let the three readings taken in the order indicated be called x, y, z. Then $y - x$ and $y - z$ are proportional respectively to the weight of the body and the weight of the water displaced by the body, and hence the specific gravity is

$$\frac{y - x}{y - z}$$

when referred to water at the temperature of the experiment.

Enter results thus :—

EXPERIMENT No. 4. III.

First reading	$x = \ldots$
Second "	$y = \ldots$
Third "	$z = \ldots$

Specific gravity $= \frac{\ldots}{\ldots} = \ldots$

IV.—To find the specific gravity of a liquid. Remove the pans from the spring, hang them up carefully, and attach the small glass sinker to the spring. Read the scale (1) when the sinker is in air, (2) when immersed in the given liquid, (3) when immersed at about the same depth in distilled water. The three readings in the order mentioned being x, y, z, the specific gravity of the liquid is

$$\frac{x - y}{x - z}$$

when referred to water at the temperature of the experiment.

Enter results thus :—

EXPERIMENT No. 4. IV.

First reading	$x = \ldots$
Second "	$y = \ldots$
Third "	$z = \ldots$

$\therefore$ Specific gravity $= \frac{\ldots}{\ldots} = \ldots$

(Each student to sign his name.)

5. MOHR'S SPECIFIC GRAVITY BALANCE.

EXPERIMENT.—To find the specific gravity of a liquid by means of Mohr's Balance.

Apparatus, etc.—Mohr's Specific Gravity Balance. A given liquid whose specific gravity is required.

Method.—Set up the balance by placing the beam on the stand. Attach the float (a mercury thermometer) to the hook of the beam, allowing it to hang freely in the jar. The beam is now in equilibrium. Turn the levelling screw until the horizontal pointer of the beam and that of the stand are exactly opposite each other, or until the former oscillates equally on each side of the latter. Riders of different masses are provided, each size $\frac{1}{10}$ of the mass of the next larger. The largest of these will, if placed on the hook, balance the beam when the float is immersed in distilled water at a temperature of $15°$ C. Hence, the weight of the rider is equal to the weight of water at this temperature which would be displaced by the float. Now immerse the float in the given liquid (pour the liquid from the bottle into the jar). Remove any air bubbles which are seen to be attached to the float. Balance the beam by placing riders at the graduations of the beam, one or both of the heaviest being (if necessary) on the hook.

If the liquid be lighter than water, the first decimal figure in the specific gravity will be the graduation occupied by the largest rider, the second that occupied by the rider of the next size, etc. If the liquid be heavier than water, begin by putting the largest rider on the hook.

Enter results as follows (calling the riders of sizes I, II, III, IV, I being the largest) :—

EXPERIMENT No. 5.

At graduation	1,	rider of size	. . .
" "	2,	" " "	. . .
" "	3,	" " "	. . .
&c.			
On hook		" " "	. . .
Temperature of the liquid		. . .° C.	

∴ specific gravity of the liquid at temperature . . . referred to water at 15° C. is . . .

Specific gravity of water at 15° is . . . (refer to table).

∴ specific gravity of the given liquid at temperature . . . referred to water at 4° C. = . . . × . . . = . . .

(Each student to sign his name.)

6. NICHOLSON'S HYDROMETER.

EXPERIMENT.—To find the specific gravity (1) of a solid, (2) of a liquid, by means of Nicholson's Hydrometer.

Apparatus, etc.—Nicholson's Hydrometer. Distilled water. Weights. A thermometer. A crystal and a liquid whose specific gravities are required.

Method I.—To find the specific gravity of the solid.

By means of weights placed in the upper pan, sink the hydrometer in the water until the mark on the stem is brought to the surface of the water. Let the weights required for this be W_1. The weights may be considered correct when the hydrometer oscillates up and down equal distances on each side of the mark. Remove some of the weights, place the crystal in the upper pan and again balance. Let the weights now be W_2. Then $W_1 - W_2$ is the weight of the crystal. Put the crystal in the lower pan and again

balance. Let the weights now be W_3. Then $W_3 - W_2$ is the weight of the water displaced. Hence the specific gravity is

$$\frac{W_1 - W_2}{W_3 - W_2}.$$

This is the specific gravity referred to water at the temperature of the water used in the experiment. Calling the specific gravity of water at this temperature ρ we have

$$\frac{W_1 - W_2}{W_3 - W_2} \cdot \rho$$

for the specific gravity of the given body, referred to water at the temperature 4° C.

Enter results as follows :—

EXPERIMENT NO. 6. I.

$$W_1 = \ldots$$
$$W_2 = \ldots$$
$$W_3 = \ldots$$
$$\text{Temp.} = \ldots$$
$$\rho \text{ for this temp.} = \ldots$$
$$\therefore \text{ specific gravity} = \frac{W_1 - W_2}{W_3 - W_2} \cdot \rho$$
$$= \ldots$$

II.—To find the specific gravity of the liquid.

Weigh the hydrometer in an ordinary balance. Call this W_1. Put weights in the upper pan until the instrument sinks in the water to the mark. Let these weights be W_2. Then $W_1 + W_2$ is the weight of water displaced by the hydrometer. Similarly sink it in the given liquid. If W_3 be required to do this, $W_1 + W_3$ is the weight of the liquid

displaced by the hydrometer. Hence, the specific gravity required =

$$\frac{W_1 + W_3}{W_1 + W_2} \cdot \rho$$

where ρ is the specific gravity of the water at the temperature of the experiment.

Enter results as follows:—

EXPERIMENT No. 6. II.

$W_1 = \ldots$
$W_2 = \ldots$
$W_3 = \ldots$
Temp. $= \ldots$
ρ for this temp. $= \ldots$
$\therefore$ specific gravity for the observed temperature =

$$\frac{W_1 + W_3}{W_1 + W_2} \cdot \rho = \ldots$$

(Each student to sign his name.)

7. THE MICROMETER CALIPERS.

EXPERIMENT.—To find the diameters of the given sphere and cylinder, and hence to find the number of centimetres in a foot.

Apparatus. — Two screw micrometers ("Micrometer Calipers").

Method.—The pitch (interval between threads) of one of the screws is $\frac{1}{40}$ (= .025) of an inch. Tenths of inches are marked on the spindle, each additional division being .025, .050, or .075 of an inch. Twenty-fifths of a turn of the screw (corresponding to $\frac{1}{1000}$ ths of an inch) are marked

on the thimble. The pitch of the other screw is $\frac{1}{2}$ of a millimetre; the thimble is divided into 50 parts, hence each division corresponds to $\frac{1}{1000}$ of a centimetre. In each instrument tenths of the thimble divisions are to be estimated. Each tenth thus corresponds to .0001 in. in one, and to .0001 cm. in the other.

Turn the milled head of the screw until the end of the screw is in contact with the end of the fixed cylinder. Always turn the screw gently so as to avoid straining the thread when contact takes place. The sense of touch will inform you when the contact is complete; do not turn any further. The micrometer should now read 0 exactly; probably it will not; determine the error by taking the mean of several settings, and afterwards apply this correction to your readings.

Now measure the diameters of the given objects with each instrument, each result being the mean of at least five settings and readings. From these results deduce the number of centimetres in a foot.

Enter results as follows :—

EXPERIMENT No. 7.

Diameter of cylinder . . . in. (mean of . . . readings)
Index correction . . . in. (" " . . . ")
Corrected value . . . in.

Diameter of cylinder . . . cm. (mean of . . . readings)
Index correction . . . cm. (" " . . . ")
Corrected value . . . cm.

Hence the number of centimetres in one foot = . . .

Enter results for the sphere in the same way.

(Each student to sign his name.)

8. THE VERNIER CALIPER.

EXPERIMENT.—To measure the given cylinder.

Apparatus—A Vernier Caliper. One jaw is fixed to a graduated bar about 15 inches long; the other slides along the bar, but may be clamped in any position. The divisions on one side of the bar are fortieths of an inch. The Vernier has 25 divisions. These 25 divisions are of the same length as 24 of the bar divisions. Hence, one divi- of the Vernier = $\frac{1}{25}$ of $\frac{24}{40}$ of an inch = .024 in., while each division on the bar = .025 in. The difference is .001 in. Hence, the instrument will read to thousandths of an inch. Every four divisions on the bar = $\frac{4}{40}$ = .1 in. The division between each tenth of an inch and the next are $\frac{1}{40}$, $\frac{2}{40}$, $\frac{3}{40}$, i. e., .025, .050, .075. Turning the bar over we find a scale of centimetres. The Vernier reads to $\frac{1}{500}$th of a centimetre. Hence the Vernier readings are to be doubled and called thousandths of a centimetre.

Method. — Loosen the clamping screw and slide the movable jaw until it comes up to the fixed jaw. The Vernier should now read 0; if it does not, the error must be noted, and applied to subsequent readings as an index correction. Now separate the jaws until the distance between them is a little more than the distance to be measured. Clamp the sliding jaw, and turn the adjusting screw until the jaws just include the given object. Read the scales on one side. Turn the caliper over and read the other scales. Repeat the work two or three times, and if the readings differ take the mean.

To measure an internal diameter, close the jaws and with a micrometer caliper, or otherwise, measure the breadth of the

jaws. (It is .3000 in. = .7620 cm.) This must be added to the readings of the scales when the jaws are opened to correspond to the internal diameter.

The given cylinder is in three parts. Measure the three diameters in order, beginning with the smallest. Find also the length of each part, and the internal diameter.

Enter results thus :—

EXPERIMENT No. 8.

	Inches.	Centimetres.
External diameter (1)		
" " (2)		
" " (3)		
Length of (1)		
" " (2)		
" " (3)		
Internal diameter . . .		

(Each student to sign his name.)

9. MICROMETRIC MEASUREMENT OF DISTANCES.

EXPERIMENT—To find the distance (not exceeding one metre) between two lines on a metal bar.

Apparatus — Two microscopes with micrometer eye-pieces. A standard metre scale.

Method — The microscopes are placed so that the centres of their fields are near the marks on the given bar,

and are then securely clamped. During the remainder of the experiment the microscope stands and tubes must not be disturbed. The parallel crosshairs of each microscope are then moved so as to be equally distant from the corresponding line on the bar. Read the divided circles, and repeat the operation several times, taking the mean of the several settings. Remove the bar, and, without disturbing the microscopes, replace it by the standard metre, adjusting the latter until its divisions are in focus. Move the metre scale until a millimetre division is near the crosshairs of one of the microscopes, while the crosshairs of the other are at the same time near one of the fine divisions of the scale. Now set the crosshairs on the nearest convenient divisions of the scale and take the readings, being careful to note the whole number of turns (if any) of the screw.

It is also necessary to determine the value of one division of each of the screw heads by moving the crosshairs over the divisions of the standard scale.

Enter results thus :—

Experiment No. 9.

Given distance = . . . cm.

Error of metre scale for this distance = . . . cm.

∴ corrected distance = . . . cm.

Each micrometer division = . . . cm. microscope A.

" " " = . . . " " B.

(Each student to sign his name.)

10. THE PLANIMETER.

EXPERIMENT.—To measure a plane area.

Apparatus.—Amsler's Polar Planimeter. This instrument consists of two metal bars jointed together. The extremity of one bar is provided with a needle point which remains fixed, while the tracing point at one end of the other bar is carried all round the perimeter of the given area. Near the end of the latter bar is a little celluloid wheel whose circumference is divided into 100 parts; with a vernier, tenths of these parts are read. Complete revolutions of the wheel are recorded on a metal circle which is turned by a worm on the axis of the first wheel. It may be proved that the given area is proportional to the distance recorded by the wheel.* The area is also proportional to the distance of the tracing point from the joint. This distance can be altered, and hence the wheel made to record in different units of area.

Method.—Let the sliding bar be set at the line marked "100 sq. cm". Read the recording circles after the tracing point has been moved to a point on the curve. Keep the needle point fixed and move the tracing point round the curve from left to right, i.e., in the direction in which the hand of a clock goes round the dial. When the point has gone entirely round, read the circles again. (The reading of the metal circle must be increased by 10 if it is less than it was at first.) The difference of the readings multiplied by 100 will give the area of the curve in square centimetres.

For example, suppose that the metal circle read 5+ and the celluloid circle 672; then the first reading recorded

* For the theory of the Planimeter see Appendix I.

is 5.672. After the motion, let the metal circle read 7 + and the celluloid circle 535; the second recorded reading is 7.535.

Then $7.535 - 5.672 = 1.863$.

$\therefore$ the area is 186.3 square centimetres.

In this case each unit of revolution recorded corresponds to 100 sq. cm., but if the sliding bar be set at the mark "10 sq. in." each unit will correspond to 10 square inches. If it be set at 200 sq.′ ¼″ = 1′, each unit will correspond to 200 sq. feet on a diagram constructed on a scale of ¼ of an inch to the foot. The area should be traced out several times and the mean taken for the final result.

Enter results thus :—

EXPERIMENT NO. 10.

1. Area of given circle.

First trial, area = 100 (. . . − . . .) = . . . sq. cm.
Second " " = 100 (. . . − . . .) = . . . " "
&c.
Mean = . . . " "

2. Area of the given irregular curve.

Enter as above.

(Each student to sign his name.)

11. THE MECHANICAL INTEGRATOR.

EXPERIMENT.—To find (1) the area of a given closed curve, (2) its centre of gravity, (3) its moment of inertia.

Apparatus.—One end of a sweeping bar traces the curve while the other end is constrained to describe a straight line parallel to the groove in a long metal bar. The sweep-

ing bar carries a divided wheel W_1. The end of the bar which describes the straight line is also the centre of two arcs, the smaller of which turns a circle carrying a wheel W_2, the larger also turning a circle carrying a wheel W_3.

Method.—Having chosen in the diagram a line OX with reference to which the moment of inertia is to be found, place the two brass arms in the groove of the steel bar and move the bar until the sharp points of the arms exactly meet OX. Then place the integrator in position, and bring the tracing point to some point in the circumference of the given area. Read the three wheels. Now trace round the given curve clockwise, and again read the wheels. Let n_1, n_2, n_3, be the *changes* of reading of W_1, W_2, W_3, respectively. Let A be the area of the curve; M be the sum of the moments, with reference to OX, of the elements of the area; and let I be the moment of inertia of the area with reference to OX. Then * will

$$A = n_1,$$

$$M = \frac{3}{5} n_2,$$

$$I = n_1 - \frac{2}{5} n_3$$

The unit is one decimetre.

The height y of the centre of gravity of the area above $OX = \frac{M}{A}$, the radius of gyration $k = \sqrt{\frac{I}{A}}$, and the moment of inertia with respect to an axis through the centre of gravity and parallel to $OX = I - Ay^2 = I - \frac{M^2}{A}$

* For the theory of the Integrator see Appendix II.

To change from decimetres to inches multiply y or k by a, A by a^2, M by a^3, and I by a^4 where $a = 3.937$, the number of inches in a decimetre; $a^2 = 15.500$, $a^3 = 61.023$, $a^4 = 240.290$.

Enter results thus :—

EXPERIMENT NO. 11.

$n_1 = \ldots,\ n_2 = \ldots,\ n_3 = \ldots$

$\therefore A = \ldots,\ M = \ldots,\ I = \ldots$

$y = \ldots,\ k = \ldots$

(Each student to sign his name.)

12. MEASURES OF VOLUME.

EXPERIMENT.—To verify certain measures of capacity.

Apparatus.—A half-litre flask. A pint flask. A burette with glass float. A thermometer. A balance, water, etc.

A litre is the volume of 1 kilogram of water at 4° C., the weighing being corrected for displaced air. It was intended to be, and is very nearly indeed, equal to one cubic decimetre.

A gallon is the volume of 10 pounds of water at 62° Fah., the weighing being by means of brass weights, and *not* corrected for displaced air. The pint is one-eighth of a gallon, or the volume of 20 ounces of water. One twentieth of a pint is called a fluid ounce; each fluid ounce is divided into 8 fluid drams, and each fluid dram into 60 minims. Hence, 8×60 or 480 minims of water weigh one ounce or 437.5 grains. Thus the graduations of measures of capacity depend upon weighing.

Method I.—Taking rather more than a pint of distilled water, exhaust the air from it by means of the air pump. See that the half-litre flask is quite dry, both inside and outside, and weigh it in balance E or F. Fill it with water up to the mark, wipe the outside if necessary, and weigh again. The difference of the two weighings, corrected for displaced air, is the weight of the water. Dividing by the specific gravity of the water at the temperature of that used in the experiment, we have the weight of the flaskful of water at 4°. This should, of course, be 500 grammes.

Enter results thus :—

EXPERIMENT No. 12. I.

Apparent weight of empty flask	= . . . grammes.
" " " flask and water	= . . . "
Difference	= . . . "
Correction for displaced air	= . . . "
Corrected difference	= . . . "
Temperature of the water	= . . . ° C.
Spec. gravity of water at this temp.	= . . .

∴ weight of flaskful of water at 4° $= \frac{\cdot\ \cdot\ \cdot}{\cdot\ \cdot\ \cdot}$ = . . . grammes.

∴ volume of flask = ... ccm.

II.—Proceed similarly with the pint flask, using grain weights. The weighings are not to be corrected for displaced air; and, as the temperature will be about 62°, no temperature correction will be required.

Enter results thus :—

EXPERIMENT No. 12. II.

Weight of empty flask	= . . . grains.
" " filled "	= . . . "
∴ weight of water	= . . . "
	= . . . ounces.

III.—To verify the graduations of the burette.

Nearly fill the tube with water. The mark on the float will serve as an index in reading the scale. Draw off water until the mark is below the top of the scale. Then draw off about 10 cc. of water in a vessel which has previously been weighed, and find the weight of the water. Read the scale in cubic centimetres and tenths, and find the weight in grammes and tenths. Continue to draw off about 10 cc. at a time until the float reaches the bottom of the scale, weighing each time.

As the results are approximate only, it will not be necessary to correct for temperature or displaced air.

Enter results thus :—

EXPERIMENT NO. 12. III.

Scale Readings.	Difference.	Weight of water.
1. ......cc. and cc	......cc.	...... grammes
2. ......" " ..... "	...... "	...... "
3. ......" " ..... "	...... "	...... "
4. ......" " ..... "	...... "	...... "
5. ......" " ..... "	...... "	...... "

IV.—A drop of water is supposed to be about a minim. Ascertain the truth of this by counting out a certain number of drops, finding the weight in grains and multiplying by $\frac{480}{437 \cdot 5}$ to reduce to minims.

Enter results thus:—

EXPERIMENT NO. 12. IV.

Weight of . . . drops = . . . grains.
∴ volume of . . . " = . . . minims.

(Each student to sign his name.)

13. ATWOOD'S MACHINE.

EXPERIMENT.—To verify the first and second laws of motion.

Apparatus.—Atwood's Machine, electric attachments, clock and chronograph.

The machine consists of a light aluminium pulley, whose axis rests on friction wheels also of aluminium. Over the pulley passes a silk threa d carrying two weights ; the one on the left we shall call W_l and the other W_r. The masses are each 100 grammes. On the weights may be placed brass discs and bars of 5 and 10 grammes mass. Two rings are attached to the scale, the upper one serves the purpose of stopping a bar placed on the top of the mass W_l, while the mass itself moves onward through the ring; the lower one is arranged so that W_l in passing through it breaks the electric circuit.

The chronograph is an instrument by which the exact instant when any event occurs may be recorded on paper. An electro-magnet attracts its armature whenever the electric circuit is closed; a little steel disc which revolves in ink is by the movement of the armature pressed against a strip of paper, which is moved along by a train of wheels actuated by a spring. One electric circuit passes through the electro-

magnet of the chronograph, and also through the clock pendulum, and is completed each second when the end of the pendulum passes through a drop of mercury. The other circuit passes through the electro-magnet of the chronograph and also through the electro-magnet and left hand key of the machine. Hence, when the key is closed, the platform supporting W_l is dropped by the movement of the armature of the electro-magnet of the machine, and at the same time a record is made on the paper. Thus by comparing this record with that corresponding to the seconds of the clock, the time at which the key is closed can be read fom the paper.

A third circuit, completed by the right hand key, puts a brake on the wheel of the machine, thus checking the motion of the weights.

(The arrangement of the wires should be carefully studied before commencing the experiment.)

Method I.—To verify the first law of motion.

Place the upper ring at any position near the top of the scale (say 50 centimetres from the top) and the break-circuit ring a little (say 20 centimetres) below it. Bring up W_l above the upper ring and place on it a bar (say the smaller one) and then pull W_l up carefully to the top of the scale, adjust the platform and let W_l come down gently on it. *At this point in the experiment and in all similar cases bring W_r to rest.* See that the contact points of the lower ring are together. Start the chronograph.

Everything now being ready make the experiment as follows :—Close the left hand key, *keeping the key pressed* until W_l has passed through the lower ring. When this occurs, immediately press the other key, thus checking the

motion of the weights; bring W_l up to the top, adjust the platform as before, and stop the chronograph.

The continuous line on the paper was made during the time that the weight was falling to the lower ring.

With a pair of dividers, compare this time with the scale of seconds on the paper and express the time in seconds and tenths. Let it be called t_1. Now lower the lower ring by any small amount (for convenience the same distance, 20 centimetres, as was taken between the two weights at the beginning) and proceed as before, getting t_2. Again lower by the same amount as before, getting t_3, and continue to lower until W_l has been placed in at least 5 positions. Then find $t_5 - t_4$, $t_4 - t_3$, $t_3 - t_2$, $t_2 - t_1$. If these are equal we shall have shown that a body will, when acted on by no force or by forces in equilibrium, move over equal distances in equal times.

Enter results thus :—

EXPERIMENT NO. 13. I.

Distance s of upper ring from 0 of scale $= \ldots$ cm.

" d by which other ring is lowered each time $= \ldots$ cm.

$t_1 = \ldots$	$t_2 - t_1 = \ldots$
$t_2 = \ldots$	$t_3 - t_2 = \ldots$
$t_3 = \ldots$	$t_4 - t_3 = \ldots$
$t_4 = \ldots$	$t_5 - t_4 = \ldots$
$t_5 = \ldots$	Sum of these $= \ldots$
	Mean $= \ldots$

Speed v after falling distance $s = \dfrac{d}{\text{above mean}} = \dfrac{\ldots}{\ldots} = \ldots$

Time t in which this speed is acquired $= \ldots$

II.—To show that the acceleration is constant, or in other words, that the speed is proportional to the time during which the force acts.

We have the speed v acquired in the time t. Place the upper ring in some other position, and as in *I* find the speed V and the time T of falling to the new position. (For this purpose the lower ring need be placed in two or three positions only.)

Then $\frac{V}{v}$ and $\frac{T}{t}$ should be equal.

Enter results thus :

EXPERIMENT NO. 13. II.

$v = \ldots$	$V = \ldots$
$t = \ldots$	$T = \ldots$
$\frac{V}{v} = \ldots$	$\frac{T}{t} = \ldots$

III.—Remove the bar. The weights are now balanced. Place a 10 gramme disc on W_l and two 5 gramme discs on W_r.

The masses are still balanced and their sum is 220 grammes. Now remove a 5 gramme disc from W_r to W_l. The sum is the same as before but the difference is 10 grammes. The 105 on W_r balances 105 on W_l, but the whole 220 will be set in motion by the attraction of the earth on the 10 grammes.

Find t_1 the time of falling through any distance s. The acceleration is $\frac{2s}{t_1^2}$.

Now move the remaining 5 gramme disc to the left. The sum of the masses is the same as before, but their difference is twice as great. Find t_2 the new time of falling through s. The acceleration is now $\frac{2s}{t_2^2}$. If this is about twice as great as $\frac{2s}{t_1^2}$, we shall have shown that the acceleration is proportional to the force when the mass remains the same, or that the rate of change of momentum is proportional to the force which produces it.

Enter results thus:

EXPERIMENT No. 13. III.

$s = \ldots$ cm. $\qquad \frac{2s}{t_1^2} = \ldots$

$t_1 = \ldots$ sec.

$t_2 = \ldots$ sec. $\qquad \frac{2s}{t_2^2} = \ldots$

$\therefore$ first acceleration $\times\ 2 = \ldots$

second " $= \ldots$

IV.—The value of g.

Let the acceleration produced by the weight of 10 grammes acting on 220 be called a_1. Then g, the acceleration produced by the weight of 220 grammes acting on 220 grammes will be $22a_1$.

Similarly $g = 11a_2$, where a_2 is the acceleration produced by the weight of 20 grammes acting on 220 grammes.

The resulting value of g is too small, since the friction retards the motion. The true value of g is about 981.

Hence, $\frac{981 - 22a_1}{981}$ is the fraction of the weight of 10 grammes, which goes to overcome the inertia of the pulleys, the friction, and resistance of the atmosphere.

This fraction may similarly be determined from a_2.

Enter results thus:—

EXPERIMENT No. 13. IV.

$$\frac{981 - 22a_1}{981} = \ldots \qquad \frac{981 - 11a_2}{981} = \ldots$$

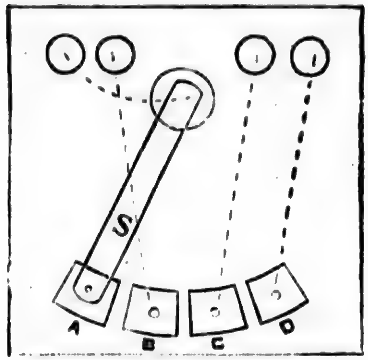
(Each student to sign his name.)

14. ATWOOD'S MACHINE.

EXPERIMENT.—To verify the first and second laws of motion.

Apparatus.—Atwood's Machine (with wooden scale), electric attachments and water clock.

The machine consists of a light aluminium pulley whose axis rests on friction wheels also of aluminium. Over the pulley passes a silk thread carrying two weights; the one on the left we shall call W_l and the other W_r. The mass of W_l is 240 and that of W_r 225 grammes. On the weights may be placed brass discs and bars of 5 and 10 grammes mass. Two rings are attached to the scale; one of these serves the purpose of stopping a bar placed on the top of the mass W_l while the mass itself moves onward through the ring; the other (the "break-circuit" ring) is arranged so that W_l in passing through it breaks the electric circuit. The binding screws at the top of the switch-board are connected by wires behind the board (as indicated by the dotted lines) with the swinging arm S and the contact pieces B, C, D, to which S may be moved.



When S is moved to B the circuit is completed, but the machine is in no way affected. When it is moved to C the current passes through the electro-magnet of the machine and also that of the water clock; hence the platform which supports W_1 drops at the same instant that the water clock is opened. When S is moved to D the weights are arrested. (The arrangement of the wires should be carefully studied before commencing the experiment.)

Method I.—Taking as the unit of time the time in which 10 c. c. of water falls into the graduated vessel, we must first find the equivalent of this in seconds.

Balance the weights (unless they are already balanced), and set the water running slowly into the cistern. Place under the cistern a flask which will contain a considerable quantity of water (say 500 c. c.). Move S to B. Then move to C, noting carefully the time. When the flask is filled to the mark, move S quickly back to A, at the same time noting the time. From this find the time in which 10 c. c. will fall.

Enter results thus :—

EXPERIMENT No. 14. I.

Time of . . . c.c. . . . sec., first trial.
" " . . . " . . . " , second trial.
" " . . . " . . . " , third trial.
Mean of times . . . "

∴ 10 c.c. is equivalent to . . . sec.

II.—To verify the first law of motion.

Place the ordinary ring at any position near the top of the scale (say 50 cm.) and the break-circuit ring a little (say 20 cm.) below it. The weights being still balanced, bring up W_l above the upper ring and place on it a bar (say the larger one), and then pull W_l up carefully to the top of the scale, adjust the platform and let W_l come down gently on it. *At this point in the experiment and in all similar cases bring W_r to rest.* See that the contact points of the break-circuit ring are together. Move S to B and then to C. *Do not remove the hand from S; watch the weight and as soon as it breaks the circuit by passing through the lower ring, immediately move S to D.* This will stop the motion. Take hold of the string on the right, move S back to A, then pull W_l up again and arrange as before. Now read the water clock and repeat the work. Let the mean of the two water times be t_1, 10 c.c. being taken for unit of time. Now lower the break-circuit ring any convenient amount, say 50 cm., and proceed as before, getting t_2. Again lower by the same amount as before, getting t_3; and continue to lower until W_l has been placed in say 5 positions. Then find $t_5 - t_4$, $t_4 - t_3$, $t_3 - t_2$, $t_2 - t_1$. If these are equal we shall have shown that a body will, when acted on by no force, move over equal distances in equal times.

Observe that all distances on the scale are to be considered with reference to the bottom of W_1.

Enter results thus :—

EXPERIMENT No. 14. II.

Distance s of upper ring from 0 of scale . . . cm.

" d by which lower ring is lowered each time . . . cm.

$t_1 = \ldots$	$t_2 - t_1 = \ldots$
$t_2 = \ldots$	$t_3 - t_2 = \ldots$
$t_3 = \ldots$	$t_4 - t_3 = \ldots$
$t_4 = \ldots$	$t_5 - t_4 = \ldots$
$t_5 = \ldots$	Sum of these = . . .
	Mean = . . . = . . . sec.

Speed v after falling distance $s = \dfrac{d}{\text{above mean}} = \dfrac{\ldots}{\ldots} = \ldots$

Time t of acquiring this speed = . . . sec.

III.—To show that the acceleration is constant, or in other words, that the speed is proportional to the time during which the force acts.

We have the speed v acquired in the time t. Place the upper ring in some other position, and as in II., find the speed V and time T of falling to the new position. (For this purpose the lower ring need be placed in two or three positions only.) Then $\dfrac{V}{v}$ and $\dfrac{T}{t}$ should be equal.

Enter results thus :—

EXPERIMENT No. 14. III.

$v = \ldots$	$V = \ldots$
$t = \ldots$	$T = \ldots$
$\dfrac{V}{v} = \dfrac{\ldots}{\ldots} = \ldots$	$\dfrac{T}{t} = \dfrac{\ldots}{\ldots} = \ldots$

IV.—In II. the whole mass attached to the two ends of the string(the bar included) is set in motion by the attraction of the earth on the bar. To determine the effects of different forces when acting on the same mass we might take a five gramme disc from the right hand side and put it on the left. Tne whole mass set in motion would remain as in II, but the moving force would be twice as great. As in II. we might find tbe acceleration which should be twice that of II. It will be simpler, however, to proceed as follows :—

Remove the bar. The weights are now balanced and their sum is 480 grammes. Change a 5-gramme disc from right to left. The sum is still 480 grammes, but the difference is 10. Put the break-circuit ring at any convenient reading s, say near the middle of the scale. Find t, the time of falling through the distance s. The acceleration is $\frac{2s}{t_1{}^2}$.

Return the 5-gramme disc to the right and move the 10-gramme disc to the left. The sum of the masses is the same as before, but their difference is twice as great. Find t_2, the new time of falling through s. The acceleration is now $\frac{2s}{t_2{}^2}$. If this is about twice as great as $\frac{2s}{t_1{}^2}$, we shall have shown that the acceleration is proportional to the force when the mass remains unchanged, or that the rate of change of momentum is proportional to the force which produces it.

Enter results thus :—

EXPERIMENT NO. 14. IV.

$s = \ldots$ cm.	$\frac{2s}{t_1{}^2} = \ldots$
$t_1 = \ldots = \ldots$ sec.	
$t_2 = \ldots = \ldots$ sec.	$\frac{2s}{t_2{}^2} = \ldots$

$\therefore$ first acceleration $\times 2 = \ldots$
second " $= \ldots$

V.—The value of g.

Let the acceleration produced by the weight of 10 grammes acting on 480 be called a_1. Then g, the acceleration produced by the weight of 480 grammes acting on 480 grammes, will be $48a_1$. Similarly $g = 24a_2$, where a_2 is the acceleration produced by the weight of 20 grammes acting on 480 grammes. The resulting value of g is too small since the friction, etc., retards the motion. The true value of g is about 981. Hence, $\frac{981 - 48a_1}{981}$ or $\frac{981 - 24a_2}{981}$ is the fraction of the acting force which goes to overcome the inertia of the pulleys, the friction and resistance of the atmosphere.

Enter results thus:—

EXPERIMENT NO. 14. V.

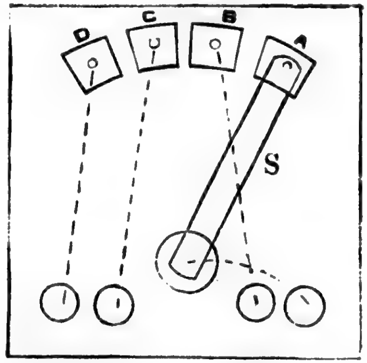
$$\frac{981 - 48a_1}{981} = \ldots \qquad \frac{981 - 24a_2}{981} = \ldots$$

(Each student to sign his name.)

15. THE INCLINED PLANE.

EXPERIMENT.—To investigate the motion of a sphere rolling down an inclined plane.

Apparatus.—An inclined plane about 3 metres in length, with electric attachments. Ivory and other balls. A water clock. A litre flask.



The binding screws at the bottom of the switch-board are connected by wires behind the board (as indicated by the dotted lines) with the swinging arm *S* and the contact pieces *B*, *C*, *D*, to which *S* may be moved. When *S* is moved to *B* the circuit is completed, but the machine is in no way affected. When it is moved to *C* the current passes down one of the wires inside the groove of the inclined plane to the bridge circuit-maker (previously placed so as to connect the wires at any desired position on the scale), thence back by the other wire in the groove, through the electro-magnet of the inclined plane, and through the electro-magnet of the water clock. Thus the armatures of the two electro-magnets are simultaneously moved, the water clock is opened at the instant that the ball starts to roll down the plane, and continues open until the ball displaces the circuit-maker connecting the wires in the groove. In this way we obtain a measure of the time required by the ball to roll through any distance on the scale.

(The arrangement of the wires should be carefully studied before commencing the experiment.)

Method I.—Choosing as a unit of time, the time during which 10 c.c. of water falls from the cistern, to find the equivalent of this time in seconds. Set the water running slowly into the cistern. Open the valve of the cistern with the electric current and note by means of the stop-watch or otherwise, the number of seconds required for 1000 c.c. of water to run out. One hundreth of this time gives in seconds the equivalent of the water clock unit. The operation should be repeated two or three times and the mean taken.

Enter results thus :—

EXPERIMENT NO. 15. I.

First trial,	1000 c.c.	in	. . .	seconds
Second trial,	"	"	"	. . .	"
Third trial,	"	"	"	. . .	"
Mean value,	"	"	"	. . .	"

∴ time of 10 c.c. = . . . seconds.

II.—To show that the time of motion over any distance is independent of the size or mass of the sphere.

Set the circuit-maker at any position on the scale and find the time of motion with one of the balls, taking as the unit of time the time in which 10 c.c. of water falls from the water clock. Keeping the distance and inclination unchanged, find the time of motion of the other balls.

Enter results thus :—

EXPERIMENT NO. 15. II.

Distance on the plane	. . . cm.
Inclination of the plane	. . . degrees.
Time of ivory ball	. . .
" " wooden ball	. . .
" " 'composition' ball	. . .

III.—Set the circuit-maker at various positions (at least 4), and show that the distance is proportional to the square of the time.

Enter results thus :—

EXPERIMENT NO. 15. III.

s (dist. on sca'e in cm.)	T (water time.)	t (seconds.)	$\frac{2s}{t^2} = a$

∴ mean value of the acceleration a for the inclination . . .° is . . . cm. per second in each second.

IV.—Change the inclination, repeat the last experiment, and show that the acceleration is proportional to the sine of the angle of inclination of the plane.

Enter results thus :—

EXPERIMENT NO. 15. IV.

s	T	t	$\frac{2s}{t^2} = a$

∴ mean value of the acceleration a for the inclination . . . is . . . cm. per second in each second.

Ratio to value in III. $= \frac{\dots}{\dots} = \dots$

Ratio of sines of angles of inclination $= \frac{\dots}{\dots} = \dots$

V.—Assuming the theoretical value of the acceleration to be

$$\frac{5}{7} g \sin \theta,$$

find both from III. and IV. the value of g.

Enter results thus :—

EXPERIMENT NO. 15. V.

From III.	$g = \dots$ cm.	per	sec.	in	each	second.
From IV.	$g = \dots$ "	"	"	"	"	"
Mean,	$g = \dots$ "	"	"	"	"	"
Hence,	$g = \dots$ ft.	"	"	"	"	"

(Results of the five parts to be returned together.)

(Each student to sign his name.)

16. CENTRIFUGAL FORCE.

EXPERIMENT.—To investigate the stress in uniform circular motion.

Apparatus.—At the centre of a horizontal rotatory table is a chamber containing mercury communicating with a vertical mercury column in a glass tube alongside a fixed scale. One side of the mercury chamber is a thin steel corrugaged diaphragm, to the centre of which is attached a radial brass rod, the other end of which is attached to a

rocking arm fastened to the re[illegible]g table. When rotation takes place the stress be[illegible] the revolving mass and the steel diaphragm causes [illegible]ncrease in the capacity of the mercury box and a corresponding fall in the mercury column. An overhead conical countershaft permits of large variations in the rate of turning of the table. We may thus investigate the connection between the mass, velocity and radius in circular motion.

Method I.—The first thing to be done is the calibration of the glass tube. By a spring balance find the reading of the top of the mercury column for various forces applied at the end of the radial bar. Plot these on section paper, the readings of the column being taken as abscissas and the corresponding forces as ordinates, and draw a curve through the points.

II.—Keep the rate of turning unchanged. Also keep the position of the sliding body unchanged, but change the mass of this body by gradual addition or subtraction, being careful to clamp it securely after each change. Plot the mass thus added or subtracted, and also the corresponding forces, and draw a line through the points.

III.—Keep the rate of turning unchanged, and also the mass, but move the latter to various positions along the rod, being careful to clamp it securely in each position. Plot the various distances from the centre and the corresponding forces, and draw a line through the points.

IV.—Keep the mass and the distance from the centre unchanged, but vary the rate of turning. The number of revolutions per minute may be noted by means of a speed indicator. Plot these and the corresponding forces, and draw a line through the points.

From an inspection of the diagrams obtained in II, III and IV. state the relation between the force and the mass, radius and speed.

Enter results thus :—

EXPERIMENT No. 16.

(The four diagrams.)

(The general statement as to results.)

17. STATICAL FRICTION.

EXPERIMENT.—To find the relation between the normal pressure and the corresponding statical friction.

Apparatus.—A spring balance. Weights. Surfaces between which the friction is to be investigated.

Method.—Keep a record of the various pressures between the rubbing surfaces and also of the force required to just overcome the friction in each case. This force must be applied very gradually and in a direction parallel to the surfaces under consideration. Then plot on section paper the corresponding pressures and frictions, the former being taken as abscissas and the latter as ordinates. Draw a straight line which will pass nearest to the various points thus obtained.

Enter results thus :—

EXPERIMENT No. 17.

(The diagram.)

(Each student to sign his name.)

18. MECHANICAL EQUIVALENT OF HEAT.

Experiment.—To determine the mechanical equivalent of heat.

Apparatus.—A steel cone containing mercury in which the bulb of a thermometer dips is caused to revolve in a similar fixed cone. Means are provided for measuring the amount of work done in heating the cones and the mercury by a measurable amount. For correction for radiation, and for other details of the experiment, see Pickering's *Physical Manipulation.*

19. TORSION AND MOMENTS OF INERTIA.

Part I.

Experiment.—A metal bar is suspended by means of a wire, the upper end of which is clamped. The direction of the wire passes through the centre of gravity of the bar, which makes horizontal oscillations under the torsional reaction of the wire. It is required to find the moment of inertia of the bar.

Let k be the couple which would twist the free end of the wire through a unit angle (one radian). The couple which twists it through the angle θ is $k\theta$. When the wire is thus twisted it exerts, as an opposite reaction, a couple $k\theta$. Thus, when the bar is swinging horizontally and makes an angle θ with its position of equilibrium, the moments of the external forces about the axis of motion $= -k\theta$ (minus since the couple tends to diminish θ). Let $I =$ the moment of inertia of the oscillating body about the axis of

Vibration = $\frac{1}{2}$ complete oscillation

rotation, and a = the angular acceleration. Then

$$I a = - k \theta \qquad (1)$$

or,

$$a = - \frac{k}{I} \theta \qquad (2)$$

Let r = the distance of any point in the moving body from the axis, and multiply both sides of (2) by r. Then

$$r a = - \frac{k}{I} r \theta$$

In this equation $r a$ is the tangential acceleration of the point, and $r \theta$ is the displacement. Hence the motion of the point is a simple harmonic motion and the time of a vibration is $\pi \sqrt{\frac{I}{k}}$. Calling this time t we have

$$t = \pi \sqrt{\frac{I}{k}}$$

$$\therefore I = \frac{k t^2}{\pi^2} \qquad (3)$$

Two bodies of equal mass m are arranged to slide along the bar. Let I_1 = the moment of inertia of the bar alone about the axis of rotation, I_2 that of each sliding mass about a vertical axis through its centre of gravity. Then a being the distance of this axis from the axis of rotation, the moment of inertia of each of the sliding masses with reference to the axis of rotation is $I_2 + ma^2$.

Method I.—The experiment is to be conducted as follows :—

1. The sliding masses are placed at distance a from the axis of rotation, the time of vibration being t_1.

2. The sliding masses are next placed at a distance b, the time being t_2.

3 They are entirely removed, the time being now t_3.

Then $$I_1 + 2I_2 + 2ma^2 = \frac{k\,t_1^2}{\pi^2} \qquad (4)$$

$$I_1 + 2I_2 + 2mb^2 = \frac{k\,t_2^2}{\pi^2} \qquad (5)$$

$$I_1 = \frac{k\,t_3^2}{\pi^2}. \qquad (6)$$

Subtracting (5) from (4),

$$2m\,(a^2 - b^2) = \frac{k}{\pi^2}\,(t_1^2 - t_2^2)$$

$$\therefore\ k = \frac{2m\pi^2\,(a^2 - b^2)}{t_1^2 - t_2^2}. \qquad (7)$$

Then from (6)

$$I_1 = \frac{2mt_3{}^2\,(a^2 - b^2)}{t_1^2 - t_2^2}. \qquad (8)$$

I_2 may be found by substituting in (4) or (5).

dynes k will be in force-distance units, and I_1 in mass-(distance)^2 units.

If I_2 were known at first, equation (5) would not be required.

k × length of wire is the couple required to twist unit length of the wire through one radian. This is the modulus of torsion of the wire.

The radius of gyration of the bar about the axis of motion = $\sqrt{I_1 / \text{mass of bar}}$.

Enter results thus :—

EXPERIMENT No. 19. I.

$a = \ldots$ cm., $b = \ldots$ cm., $m = \ldots$ grammes.
$t_1 = \ldots$ sec., $t_2 = \ldots$ sec., $t_3 = \ldots$ sec.
$\therefore\ k = \ldots$ and $I_1 = \ldots$

The radius of gyration of bar $= \sqrt{\frac{\cdots}{\cdots}} = \ldots$ cm.

PART II.

A table is similarly suspended. Let I_1 = the moment of inertia of the table alone about the axis of suspension, the time of vibration being t_1. Let a body of known moment of inertia be placed on the table, so that the centre of gravity is in the same axis of suspension, and let the time of vibration now be t_2. Let t_3 be the time of vibration when a body of unknown moment of inertia I_3 is placed on the table with its centre of gravity in the line of the wire. Then

$$I_1 = \frac{k\,t_1{}^2}{\pi^2} \qquad (1)$$

$$I_1 + I_2 = \frac{k\,t_2{}^2}{\pi^2}, \qquad (2)$$

$$I_1 + I_3 = \frac{k\,t_3{}^2}{\pi^2}. \qquad (3)$$

Subtracting (1) from (2)

$$I_2 = \frac{k}{\pi^2}\,(t_2{}^2 - t_1{}^2). \qquad (4)$$

and subtracting (1) from (3)

$$I_3 = \frac{k}{\pi^2}\,(t_3{}^2 - t_1{}^2) \qquad (5)$$

$$\therefore\ I_3 = \left(\frac{t_3{}^2 - t_1{}^2}{t_2{}^2 - t_1{}^2}\right) I_2 \qquad (6)$$

k may be found from (4) and I_1 from (1).

(6) may be written

$$I_3 = \frac{I_2}{t_2{}^2 - t_1{}^2}\ t_3{}^2 - \frac{t_1{}^2\, I_2}{t_2{}^2 - t_1{}^2}\ ,$$

$\dfrac{I_2}{t_2{}^2 - t_1{}^2}$ and $\dfrac{t_1{}^2\, I_2}{t_2{}^2 - t_1{}^2}$ may be found once for all.

Then the moment of inertia of any body is of the form

$$a\,t^2 - b,$$

where a and b are constants, and t is the time of vibration when the body is placed on the table.

Find a and b for the given table and cylinder, and then find the moment of inertia of the given pulley.

Enter results thus :—

EXPERIMENT No. 19. II.

Mass of cylinder = . . . grammes,
Diameter of cylinder = . . . cm.
$\therefore$ moment of inertia I_2 = . . .
t_1 = . . . sec., t_2 = . . . sec.
Hence a = . . . ; b = . . .
Time of vibration, table and pulley, t = . . . sec.
$\therefore$ Moment of inertia of pulley = $at^2 - b$
= . . .

(Each student to sign his name.)

20. MAXWELL'S VIBRATION NEEDLE.

Experiment.—The value of k (Exp. 19) may be found with greater accuracy by using an apparatus specially designed for this purpose by Maxwell. The time of vibration may be found with great accuracy by using a reading telescope and a chronograph. For details of the experiment see Glazebrooke and Shaw's *Practical Physics*.

21. BIFILAR SUSPENSION.

Experiment.—To find the moment of inertia of a rod.

Apparatus.— A heavy brass cylindrical rod which is hung up in a horizontal position by means of two equal vertical strings, and is caused to make small horizontal oscillations about its middle point. Apparatus is also required to measure the lengths of the strings, etc., the mass of the rod and the time of a vibration.

Method.—See that the rod is properly suspended, i.e., that it is horizontal and that the strings are vertical and equal. Measure in centimetres the distance b between the parallel strings, also h, the distance from the axis of the rod to the point of support of the string. The mass m of the rod is 534.3 grammes. Cause the rod to oscillate about its middle point through a small angle and find in seconds the time t of a vibration. Then the moment of inertia of the rod about its axis of vibration is *

$$\frac{mg\, b^2\, t^2}{4\, \pi^2\, h}$$

where $g = 981$.

* See Wright's *Mechanics*, p. 314.

Enter results thus:—

EXPERIMENT No. 21.

$b = \ldots$ cm., $h = \ldots$ cm., $t = \ldots$ sec.

$\therefore$ moment of inertia $= \ldots$

(Each student to sign his name.)

22. REVERSION PENDULUM (KATER'S).

EXPERIMENT.—To find the value of g.

Apparatus.—A metal bar carries a pair of cylinders and parallel knife edges at or near its extremities. One of the cylinders is hollow, the other solid; hence, although geometrically symmetrical, the centre of gravity is not at the middle of the bar.

Let the distances of the knife-edges from the centre of gravity be h_1 and h_2 and the times of small vibrations about these knife-edges be t_1 and t_2, respectively. Also take k as the radius of gyration of the pendulum about the axis through the centre of gravity parallel to the knife-edges.

Then

$$t_1 = \pi \sqrt{\frac{k^2 + h_1{}^2}{gh_1}}, \qquad (1)$$

$$t_2 = \pi \sqrt{\frac{k^2 + h_2{}^2}{gh_2}}, \qquad (2)$$

to eliminate k and solve for g.

Squaring,
$$t_1{}^2 h_1 = \frac{\pi^2}{g}(k^2 + h_1{}^2)$$

$$t_2{}^2 h_2 = \frac{\pi^2}{g}(k^2 + h_2{}^2)$$

subtracting,

$$t_1{}^2 h_1 - t_2{}^2 h_2 = \frac{\pi^2}{g}(h_1{}^2 - h_2{}^2)$$

$$\therefore \quad \frac{\pi^2}{g} = \frac{t_1{}^2 h_1 - t_2{}^2 h_2}{h_1{}^2 - h_2{}^2} \qquad (3)$$

which is identical with

$$\frac{2\pi^2}{g} = \frac{t_1{}^2 + t_2{}^2}{h_1 + h_2} + \frac{t_1{}^2 - t_2{}^2}{h_1 - h_2}. \qquad (4)$$

(3) will give the same value of g as (4); the latter, however, shows by what arrangement the greatest accuracy may be secured. The times t_1 and t_2 may be found with great accuracy, but the exact position of the centre of gravity is not easily determined; hence, h_1 and h_2 are uncertain. The denominator $h_1 + h_2$ of the first fraction on the right of (4) is the distance between the knife-edges, and hence may be measured even if the centre of gravity were entirely unknown. The denominator $h_1 - h_2$ of the second fraction cannot be determined with very great accuracy. On account of the fact that one of the cylinders is much heavier than the other, $h_1 - h_2$ is not small. If then t_1 and t_2 are very nearly equal, the second fraction will be very small, and hence this second term will have only a very slight effect on the value of g. Thus the pendulum should be arranged so that t_1 and t_2 are as nearly equal as possible. The distance between the knife-edges may be found by Exp. 9 (or more accurately by means of a dividing engine). The approximate position of the centre of gravity may be determined by balancing the pendulum on a horizontal knife-edge. The time of vibration may be obtained by means of coincidences of vibration of the pendulum and that of the mean time clock, which beats seconds. Special

arrangements are provided for the purpose of obtaining these coincidences. If p = the number of seconds between coincidences, and if the pendulum gains as compared with the clock, the time of vibration of the pendulum $= \frac{p}{p+1}$ seconds. The value of p adopted should be the mean of a large number of determinations.

Find the value of g, arranging the work as in the following example :—

EXAMPLE.

$$t_1 = \frac{26.97}{27.97} = .96425 \text{ sec.}$$

$$t_2 = \frac{25.67}{26.67} = .96250 \text{ sec.}$$

$$t_1{}^2 = .929773, \quad t_2{}^2 = .926415,$$

$$t_1{}^2 + t_2{}^2 = 1.856188, \quad t_1{}^2 - t_2{}^2 = .003358,$$

$$h_1 + h_2 = 92.681 \text{ cm.},$$

$$h_1 = 62.01 \text{ approx.}, \quad h_2 = 30.67 \text{ approx.},$$

$$h_1 - h_2 = 31.34 \text{ approx.}$$

$$\therefore \frac{2\pi^2}{g} = \frac{1.856188}{92.681} + \frac{.003358}{31.34},$$

$$= .0200277 + .0001071,$$

$$\therefore g = 980.34,$$

Correction for arc* = .05

" " buoyancy† = .15

$\therefore$ corrected g = 980.54.

Note 1.—When the knife-edges, etc., are arranged so that t_1 and t_2 are equal, it is known that $h_1 h_2 = k^2$. Moreover, the knife-edges should be at equal distances from the

* See Note 2.

† See Note 3.

middle of the bar in order that the action of the atmosphere may be the same in both positions of the pendulum. The required adjustment may be made as follows: The knife-edges being near the ends of the bar, observe t_1 in (1) and measure h_1 approximately, also assume an approximate value of g and calculate k^2, which $= \frac{t_1{}^2 g h_1}{\pi^2} - h_1{}^2$.

Let x = the required distance of each knife-edge from the middle of the bar and let a = the distance of the centre of gravity from the middle.

Then $$(x - a)\ (x + a) = k^2$$

$$\therefore\ x = \sqrt{a^2 + k^2}$$

The shifting of the knife-edges will change k slightly, hence the work should be repeated.

Note 2.—The correction to the time for the arc of vibration is approximately $$-\frac{1}{64}\,\theta^2$$

where θ is the radian measure of the angle of vibration. Now,

$$g = \frac{\pi^2}{t^2}(h_1 + h_2), \text{ nearly}\ ;\ \therefore\ \frac{dg}{g} = -\frac{2dt}{t},\ \therefore\ dg = \frac{1}{32}\,\frac{g\,\theta_2}{t},$$

the arc being the same in both positions; or, $\frac{1}{64}\theta^2$ may be subtracted from each time.

Note 3.—The correction for buoyancy is approximately $g\frac{s_1}{s_2}$ where s_1 and s_2 are the specific gravity of the air and metal respectively. Approximately, $\frac{s_1}{s_2} = \frac{.0012}{8} = .00015$; $\therefore\ dg = .15$ approx. This correction is obtained thus:—

$$\frac{\text{wt. of displaced air}}{\text{wt. of pendulum}} = \frac{s_1}{s_2}\ \therefore \text{ wt. of displaced air} = mg\,\frac{s_1}{s_2}$$

where m = mass of pendulum. Hence, external force moving pendulum $= mg - mg\,\frac{s_1}{s_2} = mg\left(1 - \frac{s_1}{s_2}\right)$

Hence, g' obtained by calculation $= g\left(1 - \frac{s_1}{s_2}\right)$

$\therefore g = g'\left(1 - \frac{s_1}{s_2}\right)^{-1} = g'\left(1 + \frac{s_1}{s_2}\right)$, approximately.

23. SIMPLE PENDULUM (BORDA'S.)

EXPERIMENT.—To find the value of g.

Apparatus.—The so-called simple pendulum is really a compound pendulum made up of (1) the ball, (2) the knife-edge and screw, etc., (3) the wire, (4) a cylinder above the plane of suspension.

This cylinder is intended to neutralize the effect of the wire, knife-edge, etc., leaving the ball to move as if it were attached to a mere point of suspension by a massless string. That this may be the case the cylinder must be placed so that the time of vibration of itself with the wire and knife-edge may be the same as that of the complete pendulum. This may be effected by calculation (see note 5).

The equivalent length l of the pendulum is $\frac{\frac{2}{5}r^2 + h^2}{h}$ where r is the radius of the ball and h the distance from the plane of support to the centre of the ball. These are obtained by measurement with a cathetometer and a Vernier caliper. The time is obtained by coincidences with the mean time clock. The pendulum loses a little as compared with the clock. If p be the number of seconds between coincidences, the time of vibration of the pendulum is $\frac{p}{p-1}$.

Then from $t = \pi\sqrt{\frac{l}{g}}$ we have

$$g = \frac{\pi^2 l}{t^2} = l\left[\frac{\pi(p-1)}{p}\right]^2,$$

where $l = \dfrac{\frac{2}{5}r^2 + h^2}{h}$

Find the value of g, arranging the results as in the following example :

EXAMPLE.

h = 104.316 cm., r = 2.99 cm., $\therefore$ l = 104.350 cm.
p = 41.185 sec., from 200 coincidences.

$$\therefore l\left[\frac{\pi(p-1)}{p}\right]^2 = 980.49$$

Correction for arc *	$\doteq$	.04
" for buoyancy †	=	.15
$\therefore g$	=	980.86

Note 1.—From $t = \pi\sqrt{\frac{l}{g}}$ we have $g = \frac{\pi^2 l}{t^2}$

$$\therefore \frac{dg}{g} = \frac{dl}{l} - 2\frac{dt}{t}, \text{ or } dg = \frac{g}{l}\,dl - 2\frac{g}{t}\,dt.$$

The approximate values of g, l and t are 1000, 100, and 1 respectively.

$$\therefore dg = 10\,dl - 2000\,dt.$$

Hence, an error of .001 of a second in determining the time of vibration is twice as bad as an error of 1 millimetre in measuring the length.

---

* See Note 2, Exp. 22.
† " " 3, " 22.

Note 2.—Differentiating $g = l \left[\frac{\pi\,(p-1)}{p}\right]^2$, we have

$$dg = 1.2\ dp + 9.5\ dl$$

for this pendulum.
For arc, $dg = .0031\ a^2$, where a is the total arc of vibration in centimetres.

Note 3.—From $t = \frac{p}{p-1}$ we have

$$dt = -\frac{dp}{(p-1)^2} = -\frac{dp}{1600} \text{ approximately.}$$

Thus an error of 1 sec. in p would cause an error of about $\frac{1}{1600}$ sec. in the value of t.

Note 4.—To reduce the value of g to what it would be at the sea level, we have, since g is inversely proportional to the square of the distance r from the earth's centre,

$$\frac{dg}{g} = -2\frac{dr}{r}.$$

The increase is $\therefore 2\frac{g}{r}h$, where h is the height of the pendulum above sea level. This is equivalent to increasing the metric value of g by .000093 for each foot above sea level.

Note 5.—The adjustment of the cylinder was made as follows:

The time of vibration of the whole pendulum is

$$t = \frac{\pi}{\sqrt{g}}\sqrt{\frac{m_1 k_1{}^2 + m_2 k_2{}^2 + m_3 k_3{}^2 + m_4 k_4{}^2}{m_1 h_1 + m_2 h_2 + m_3 h_3 - m_4 h_4}} \quad (1)$$

m being mass, k radius of gyration about its line of suspension, and h the distance of this line from centre of

gravity ; the four parts of the numr. and denr. referring to (1) the ball, (2) the knife-edge, screw, etc., (3) the wire, (4) the cylinder.

The m's are found in the ordinary way, also h_1, h_2, h_3. For the k's we have $k_1{}^2 = h_1{}^2 + \frac{2}{5} r^2$, r being radius of ball ; k_2 may be found experimentally by letting the knife-edge and screw vibrate on the plane of suspension and noting the time of a vibration ; $k_3{}^2$ is approximately $\frac{1}{3}$ (length of wire)^2 ; $k_4{}^2 = h_4{}^2 + \frac{1}{12}$ (length of cylinder)$^2 + \frac{1}{4}$ (radius of cylinder)^2. Thus everything is known except h_4. This is to be determined so that

$$\frac{m_2 k_2{}^2 + m_3 k_3{}^2 + m_4 k_4{}^2}{m^2 h_2 + m_3 h_3 - m_4 h_4} = \frac{m_1 k_1{}^2}{m_1 h_1} \qquad (2)$$

and hence depends upon the solution of a quadratic equation. The right hand side $= h_1 + \frac{2}{5}\frac{r^2}{h_1}$. This was found by measurement to be 104.344 ; h_2 was taken as 0, $h_3 = 50.5$, $m_2 = 3$, $m_3 = 1.57$, $m_4 = 21.37$, $k_2{}^2 = 2$, $k_3{}^2 = 3500$, $k_4{}^2 = h_4{}^2 + \frac{1.}{12} + \frac{.81}{4} = h_4{}^2 + .28$. The above equation is then (approximately)

$$\frac{6 + 5500 + (21.37\, h_4{}^2 + 6)}{79.29 - 21.37\, h_4} = 104.34 \qquad (3)$$

which is very nearly satisfied by $h_4 = 1.2$.

The cylinder being thus placed we have

$$t = \frac{\pi}{\sqrt{g}} \sqrt{h_1 + \frac{2}{5}\frac{r^2}{h_1}}$$

$$= \frac{\pi}{\sqrt{980.6}} \sqrt{104.344}$$

$$= 1.0248$$

If the cylinder were omitted we should have, substituting in (1) and making $m_4 = 0$,

$$t = 1.0247$$

This would make g about .2 too great.

This was verified by experiment.

24. SIMPLE MACHINES.

EXPERIMENT.—When work is done on a machine, a part of the energy thus expended is always used up in overcoming friction; another part may be stored up in the machine, while a third part may be employed in doing work upon some outside body or bodies. It is required to determine these several parts in the case of certain simple machines, also the mechanical advantage or purchase of the machines, *i.e.*, the ratio of the force overcome to the force applied, and the efficiency of the machines, *i.e.*, the ratio of the useful work accomplished by the machines to the energy expended on them.

Begin with the Differential Wheel and Axle. When by pulling on the string the wheel is caused to turn, friction is overcome, useful work is done in raising the weight attached to the pulley, and by raising the pulley itself energy is stored up in the machine. The unit force employed in this experiment is the weight of one-quarter ounce and the unit distance may be taken as one centimetre. Note the heights of the weight attached to the pulley and that attached to the wheel; take hold of the latter and pull it down through any distance, and again take the two heights. The weight

attached to the pulley multiplied by the distance raised is the useful work done; the weight of the pulley multiplied by the same distance is the work stored up in the machine, while the work done upon the machine is the distance moved by the weight attached to the wheel multiplied by the force required to pull the string, so as to cause the motion. To determine this, attach the scale pan (weight = 1.7 qr. oz.) and in it place weights until the motion begins. Find the decimal which the energy stored up in the machine is of the work done upon the machine; let this decimal be called D. Next the decimal which the useful work is of the work done on the machine; this is the efficiency, call it E. Then $1 - (D + E)$ is the part of the applied energy which is expended on friction; call this F. Also find the mechanical advantage A.

Similarly with the systems of pulleys, marked II*a*., II*b*., I.

Also with the differential pulley. In this case the weight of the chain may be neglected, as the weights of the strings are in the other machines. The pulley to which the 56-lb. weight is attached weighs 3.3-lbs. The force required to pull the chain so as to cause motion may be found by means of a spring balance.

In the case of the screw, apply the moving force horizontally, by means of a spring balance, to the end of the lever and perpendicular to the lever. Let it be understood that the lever makes one complete revolution. The weight of the lever must be included with that of the screw

Enter results thus :—

EXPERIMENT NO. 24.

	D.	*E.*	*F.*	*A.*
Differential Wheel and Axle.				
Pulleys II*a*.				
" II*b*.				
" I.				
Differential Pulley.				
Screw.				

(Each student to sign his name.)

25. THE MERCURY BAROMETER.

EXPERIMENT.—To find the pressure of the atmosphere.

Apparatus.—A Mercury Barometer with a scale of millimetres and inches, and an attached Thermometer, giving Centigrade and Fahrenheit readings.

Method.—Read first the attached thermometer. By means of the screw at the bottom of the barometer raise or lower the mercury in the cistern until the surface just meets the ivory cone which forms the zero point of the scale. Tap the upper part of the tube slightly so that the mercury will take its proper position. The vernier and the sliding piece at the back of the tube which moves with the vernier must now be moved so that the plane joining their bottoms exactly grazes the top of the column of mercury.

Read both the scale of millimetres and the scale of inches. To ensure as great accuracy as possible, the readings finally adopted should be the means derived from several settings of the cistern and vernier.

The observed height must now be corrected so as to give the equivalent height at the freezing temperature of water. It is known by experiment that the length of a mercury column decreases by about .000181 of itself for each degree (centigrade) in the fall of temperature, and a brass tube (such as that to which the scale is attached) by about .000019 of itself. The difference of these is .000162. Hence the correction to reduce the observed height to what it would be at 0° centigrade is

$$-\ (.000162)\ h \text{ millimetres}$$

where h is the observed height in millimetres (corrected for the index error mentioned below) and t the temperature (centigrade) of the attached thermometer.

If the temperature be Fahrenheit we must take $\frac{5}{9}$ of .000162 to find the change for each degree. Hence the correction to reduce to 32° F. is

$$-\ (.00009)\ (t - 32)\ h \text{ inches}$$

where h is the observed height in inches (corrected for the index error mentioned below) and t is the temperature Fahrenheit.

There are several other corrections to be allowed for in order that the true pressure of the atmosphere may be found.

(1) There may be a little air at the top of the mercury column and there is certainly some vapour of mercury.

(2) The ivory point may not be correctly placed.

(3) Owing to capillary action between the mercury and the glass tube, the column is not so high as it would otherwise be.

These errors are more or less uncertain, but are nearly constant, and hence are best allowed for by combining them into an "index error" determined by comparison with a standard barometer. The index correction of the laboratory barometer may be assumed to be 1.2 mm., or .047 in.

Enter results as follows :—

EXPERIMENT No. 25.

Attached thermometer..................	... ° C.*	... °F.*
Correction to thermometer............	+ 1°	+ 2°
Corrected temperature.........	.....	.....
Observed height of barometer	... mm.	... in.
Index correction..............	+ 1.2 "	+.047 "
Correction to reduce to freezing temperature	− ... "	− ... "
Corrected height.............	... "	... "

The final result should not have more than four decimal places in inches and two in millimetres.

(Each student to sign his name.)

26. THE ANEROID BAROMETER.

EXPERIMENT.—To read the aneroid barometer and by it to determine a height.

Apparatus.—The aneroid barometer consists of a cylindrical box made of thin elastic metal, from which the air

*Read the thermometer to the nearest degree.

has been exhausted. A change in the pressure of the atmosphere causes the top of the box to rise or fall through a very small distance, this motion being magnified by means of levers and communicated to a hand which moves on a graduated dial. The dial has three double scales; the interior graduations corresponding to inches of a mercury barometer, and the outer to feet in a vertical height through which the barometer may be carried. The instrument is compensated for changes in temperature.

Method.—Read the scale of inches. Also read the mercury barometer and reduce to 32 F. From this subtract the reading of the aneroid. This gives the correction to apply to the aneroid readings for correct pressure at 32 F.

Carry the aneroid barometer to the highest floor of this building. Read the scale of feet. Next carry it to the lowest floor and again read the scale. From this find the distance between the floors.

Enter results as follows:—

EXPERIMENT NO. 26.

I.

Mercury barometer reduced to 32 F.	... in.
Aneroid reading	... "
∴ index correction of aneroid	... "

II.

Reading at highest floor	... ft.
" lowest "	... "
∴ distance between floors is	... "

(Each student to sign his name.)

27. BOYLE'S LAW.

EXPERIMENT.—(1) To find the pressure of the atmosphere, and (2) to verify Boyle's Law.

Apparatus.—Two glass tubes connected by a flexible tube containing mercury are mounted on each side of a vertical scale. One tube is fixed in position, and is provided with a tap near the upper end; the other, which is open at the top, slides up and down the scale, and is balanced by a counterpoise to which it is attached by means of a string which passes over a pulley at the top of the scale.

Method I.—The tap being open, slowly lower the counterpoise until the mercury just rises above the tap. Close the tap and raise the counterpoise. After raising it six or seven decimetres it will be noticed that the mercury is beginning to fall below the tap. Wait a short time for the mercury to come to rest, read in decimetres and decimals the heights of the tops of the columns on the two sides of the scale and subtract. This will give the length of the column of mercury which is supported by the pressure of the atmosphere. The work should be repeated at least two or three times.

Enter results thus:—

EXPERIMENT NO. 27. I.

	Height of left-hand column.	Height of right-hand column.	Difference.
First Trial..			
Second Trial.			
Third Trial..			

The average of these results gives ... decimetres of mercury for the pressure of the atmosphere.

II. To investigate the relation between the pressure and volume of a given quantity of air.

The counterpoise having been lowered and the tap opened, bring the mercury to about 6 on the scale, and close the tap. There is now enclosed in the left-hand tube a column of air about 4 decimetres in height and at a pressure already determined in Part I. of this experiment, and which we may call p_1. Read carefully the top of the mercury and subtract from 10.00, which may be assumed as the height of the top of the air column. Call this result V_1. This may be taken as the volume of the air column, the volume of one decimetre of the tube being assumed as unit volume.

Now lower the counterpoise about one decimetre and read the mercury columns on the two sides. The reading on the left, subtracted from 10, gives V_2, the new volume; the difference of the two readings, added to p_1, gives p_2, the new pressure.

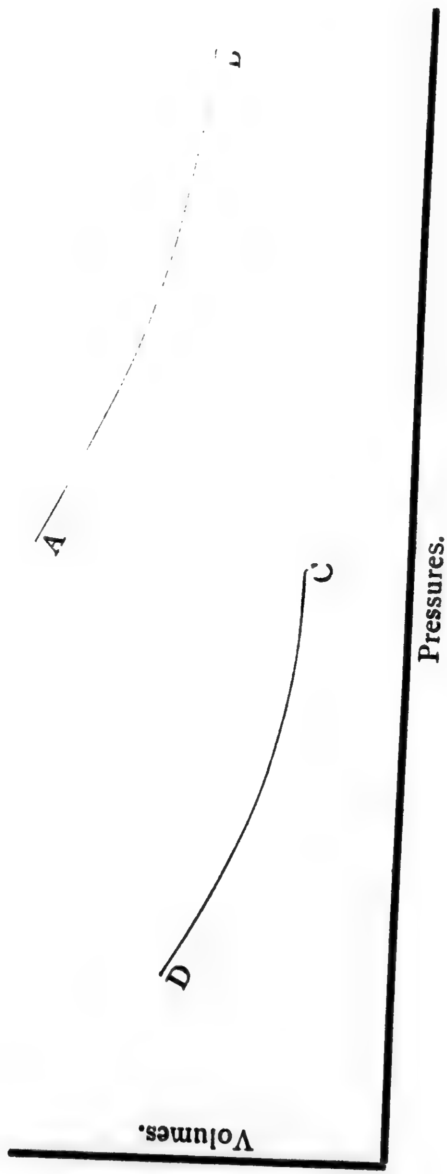
Again lower the counterpoise about another decimetre, and proceed as before; and continue until six or eight values of the volume and the corresponding pressures have been obtained. It will be found that the product of any volume by the corresponding pressure is very nearly constant, showing that the volume is inversely proportional to the pressure. This is Boyle's Law.

Having completed this, raise the counterpoise, open the tap, and enclose as before a column of air about one decimetre in length and proceed as before, but in this case allow the air to expand instead of compressing it.

Enter results thus :—

EXPERIMENT NO. 27. II.

	Pressures.	Volumes.	Product.
For Compression.			
For Expansion.			


A
C
D
Pressures.
Volumes.

III.—To express the results by means of a curve. On a piece of section paper assume one centimetre to represent, when taken horizontally, a unit of pressure; and, when taken vertically, a unit of volume. From any horizontal line cut off a length to represent the pressure of the air at any time in the experiment, and from the extremity a perpendicular line to represent the corresponding volume, thus obtaining a point A. Represent in a similar way the other pressures and volumes, and through the points thus obtained draw a curve. In this way curves similar to AB and CD will be obtained. These curves are parts of hyperbolas, having the pressure and volume lines for asymptotes.

Enter results thus:—

EXPERIMENT No. 27. III.

(Plot the curves.)

(Each student to sign his name.)

28. THE WET-BULB HYGROMETER.

(Double Interpolation.)

EXPERIMENT.—To find the pressure of the aqueous vapour in the atmosphere, also the relative humidity and the dew point.

Apparatus.—Two thermometers; the bulb of one is dry, that of the other is covered with linen which is kept moist by water which soaks up a wick.

Air at a given temperature will contain only a certain quantity of the vapour of water ; the higher the temperature, the greater the quantity it will contain. The rate at which water evaporates depends upon the ratio of the quantity of vapour actually present in the air to the total amount which the air is capable of holding. The smaller this ratio, the greater the relative dryness of the air and the more rapid the evaporation. Again, evaporation is always accompanied by a fall of temperature; the more rapid the evaporation, the greater the fall of temperature. Thus the difference between the readings of the two thermometers will indicate the rate at which evaporation is going on from the wet bulb, and hence also the relative dryness or moistness of the atmosphere. When this difference is zero no evaporation is taking place, and the air is saturated with moisture. When in this condition a slight fall of temperature will result in some of the moisture being condensed into cloud or dew.

Method.—Read the two thermometers, being careful to estimate first and quickly the number of tenths in the fraction of a degree on each scale. (If you remain near the thermometer a short time, the mercury will rise—hence the precaution just mentioned.) With the dry bulb temperature and the difference of the two, enter the following table, which will give in millimetres of mercury the pressure of the vapour actually present in the atmosphere.

Dry Bulb. Temperatures Centigrade.	Difference of Dry and Wet Bulbs. 0	1	2	3	4	5	6	7	8	9	10	11
0	4.6	3.7	2.9	2.1	1.3							
1	4.9	4.0	3.2	2.4	1.6	0.8						
2	5.3	4.4	3.4	2.7	1.9	1.0						
3	5.7	4.7	3.7	2.8	2.2	1.3						
4	6.1	5.1	4.1	3.2	2.4	1.6	0.8					
5	6.5	5.5	4.5	3.5	2.6	1.8	1.0					
6	7.0	5.9	4.9	3.9	2.9	2.0	1.1					
7	7.5	6.4	5.3	4.3	3.3	2.3	1.4	0.4				
8	8.0	6.9	5.8	4.7	3.7	2.7	1.7	0.8				
9	8.6	7.4	6.3	5.2	4.1	3.1	2.1	1.1	0.2			
10	9.2	8.0	6.8	5.7	4.6	3.5	2.5	1.5	0.5			
11	9.8	8.6	7.4	6.2	5.1	4.0	2.9	1.9	0.9			
12	10.5	9.2	8.0	6.8	5.6	4.5	3.4	2.3	1.3			
13	11.2	9.8	8.6	7.3	6.2	5.0	3.9	2.8	1.7	0.6		
14	11.9	10.6	9.2	8.0	6.7	5.6	4.4	3.3	2.2	1.1		
15	12.7	11.3	9.9	8.6	7.4	6.1	5.0	3.8	2.7	1.6	0.5	
16	13.5	12.1	10.7	9.3	8.0	6.8	5.5	4.3	3.2	2.1	1.0	
17	14.4	13.0	11.5	10.1	8.7	7.4	6.2	4.9	3.7	2.6	1.5	0.4
18	15.4	13.8	12.3	10.9	9.5	8.1	6.8	5.5	4.3	3.1	2.0	0.9
19	16.4	14.7	13.2	11.7	10.3	8.9	7.5	6.2	4.9	3.7	2.5	1.4
20	17.4	15.7	14.1	12.6	11.1	9.7	8.3	6.9	5.6	4.3	3.1	1.9
21	18.5	16.8	15.1	13.5	12.0	10.5	9.0	7.6	6.3	5.0	3.7	2.5
22	19.7	17.9	16.2	14.5	12.9	11.4	9.9	8.4	7.0	5.7	4.4	3.1
23	20.9	19.0	17.3	15.6	13.9	12.3	10.8	9.2	7.8	6.4	5.1	3.8
24	22.2	20.3	18.4	16.6	14.9	13.3	11.7	10.1	8.7	7.2	5.8	4.5
25	23.6	21.6	19.7	17.8	16.0	14.3	12.7	11.1	9.5	8.0	6.6	5.2
26	25.0	22.9	21.0	19.0	17.2	15.4	13.7	12.1	10.5	8.9	7.4	6.0
27	26.5	24.9	22.3	20.3	18.4	16.6	14.8	13.1	11.4	9.8	8.3	6.8

EXAMPLE 1.—Dry bulb 15°, wet bulb 11°. Difference $= 4^\circ$. The table gives 7.4 as the pressure.

EXAMPLE 2.—Dry bulb $16^\circ.3$, wet bulb $11^\circ.7$. Difference $= 4^\circ.6$.

$$\therefore \text{pressure} = 8.0 - (.6 \times 1.2) + (.3 \times .7) = 7.51.$$

The reading of the dry bulb is of course the temperature of the atmosphere, i.e., of the mixture of air and aqueous vapour in contact with the thermometer. Opposite this temperature and under the column headed o we find what the pressure of vapour would be if the air were saturated with moisture. For example, at 15° the saturation pressure is 12.7; at 16°.3 it is $13.5 + (.3 \times .9) = 13.77$. The ratio of the pressure of vapour actually present to the pressure at saturation gives the fraction which the vapour present is of the quantity at saturation. When this fraction is multiplied by 100 we have the percentage which the vapour present is of saturation, i.e., of the possible amount that might be present. This is called the relative humidity. Thus in Example 1, the relative humidity $= 100 \times \frac{7.4}{12.7} = 58.3$, i.e., the vapour in the air is 58.3 per cent. of the possible amount at the temperature 15°.

The dew point is the temperature to which the air would have to be cooled before the moisture actually present would begin to condense. Look out in the column headed o the pressure already found for the moisture now in the air, and opposite it, in the first column, we find the dew point.

In Example 1, the dew point is 6°.8.

Enter results as follows :—

EXPERIMENT NO. 28.

Dry bulb	...
Wet bulb	...
Difference	...
Pressure of vapour	...
Saturation pressure at dry bulb temp.	...
Relative humidity	...
Dew point	...

(Each student to sign his name.)

29. THE STEREOMETER.

EXPERIMENT.—To find the volume and specific gravity of small quantities of sugar, powder, wool, etc., which cannot be conveniently weighed in water.

An ordinary determination of specific gravity depends upon finding the weight of water displaced by the given body. By means of the stereometer the amount of air displaced by the given body may be found, and hence the the volume and specific gravity. For details see Stewart and Gee's *Practical Physics*.

APPENDIX I.

THEORY OF THE PLANIMETER.

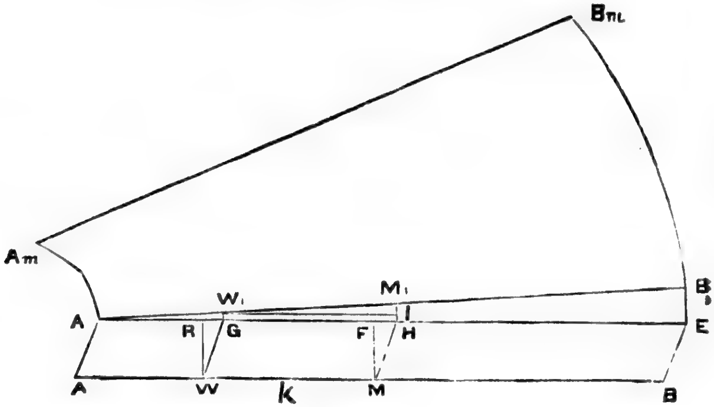


Fig. 1.

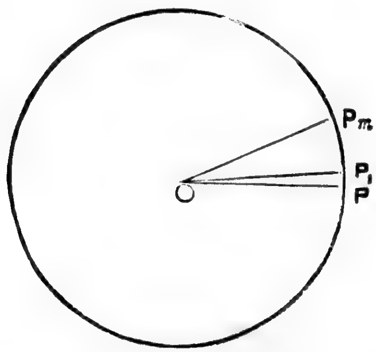


Fig. 2.

Imagine a straight line AB (Fig. 1) to be provided with a wheel placed *at its midd'e point* M so that its axis is in the direction of AB, and that suitable graduations will record as in the case of the planimeter the number of revolutions and parts of a revolution which the wheel makes. Let n stand for this number, i.e., for the change of reading of the circle between the time of starting and any subsequent time. Let c = the length of the circumference of the wheel. Then cn is the distance rolled through by a point in the circumference of the wheel.

Take b for the length of AB.

Suppose AB, starting from the position in the diagram, to be an instant later at $A_1 B_1$. This infinitesimal displacement may be considered as made up of a parallel translation to $A_1 E$, and a rotation from $A_1 E$ to $A_1 B_1$. In the first motion the wheel in moving from M to H rolls through the distance MF and slides through FH; during the second motion the wheel rolls through HM_1; hence $MF + HM_1 = cn$, where n is the change of reading of the circle. The area which the line AB has swept out is made up of AE and $A_1\ E\ B_1$. The area of AE is $AB.\ MF$, and that of A_1EB_1 is $A_1E.\ HM_1$ ($\therefore HM_1$ which may be regarded as perpendicular to A_1E is $\frac{1}{2}\ EB_1$). Hence $AB\ (MF + HM_1)$ or bcn is equal to the area ABB_1A_1 swept out by AB. Similarly if the line move forward into other positions the area which it sweeps out will continue to be the length of AB multiplied by the distance through which its middle point is displaced perpendicularly to the moving line, i.e., the distance over which the wheel rolls. In other words the area described up to any instant is bcn, where n is the change of reading of the circle up to that instant.

In order that this result may hold for all possible movements of the line, it is necessary to consider that the area, or a certain portion of it, is in certain cases negative. Let us agree to consider the area formed by all parts of the moving line which, as we look from A towards B, are advancing towards the left as $+$, and the area formed by those parts which are advancing towards the right as $-$. Then, whatever the motion of the line in the plane, the algebraic sum of the $+$ and $-$ areas swept out is equal to bcn, where n is the final change of reading of the recording circle.

Suppose, for example, that AB (Fig. 3) moves into the position A_1 B_1 by turning about C. Then in accordance with the distinction just made BCB_1 is $+$ and ACA_1 is $-$. The sum of these (i.e., the arithmetical difference) is equal

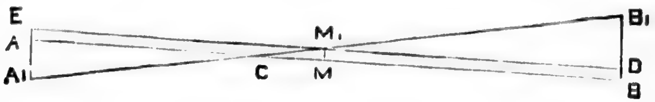


Fig. 3.

to the product of AB by the perpendicular displacement MM_1 of its middle point. For the triangles $DM_1 B_1$ and $A_1 M_1 E$ being equal, $BCB_1 = BE + ACA_1$ or $BCB_1 - ACA_1 = BE = AB.MM_1 = bcn$.

Consider now the effect of putting the wheel at any point W in AB (Fig. 1). When AB moves to $A_1 B_1$ the wheel will roll through WR and GW_1. Hence, cn being the distance rolled over by the wheel, to get the area swept out by AB we must add $b \times IM_1$ to bcn, $W_1 I$ being parallel to $A_1 E$ or AB. Draw (Fig. 2) a circle with WM or $W_1 I$ as radius, and from its centre O draw OP and OP_1 parallel to AB and $A_1 B_1$. Then $PP_1 = IM_1$.

Hence, the area swept out by $AB = bcn + b. PP_1$; and if AB move into any new position $A_m B_m$ to which OP_m is parallel, the resultant area swept out by $AB = bcn + b$. arc PP_m . If AB move about, turn backward, and finally return to the position AB or parallel to it ; the area bcn will require no correction, and the wheel will read exactly as if it were at M. But if AB make a complete revolution and return to its first position, or parallel to it, we must correct bcn by adding $b \times$ circumf. of the circle Fig. 2, i.e., by adding $2\pi bk$ where $k = WM$, the distance of the wheel from the middle point of the moving line.

Let any straight line move so that its extremities describe any closed curves. Then in all cases the area swept out by the line will be equal to the arithmetical difference of the areas of the curves described by its extremities.

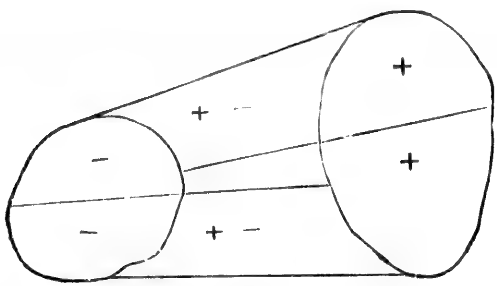


Fig. 4.

When the areas are without one another, one will be described on the whole positively and the other on the whole negatively, while the area between the two which is swept out at all will be swept out both positively and negatively. When they intersect, the common portion, in so far as it is swept at all, will be swept both positively and negatively ; the rest as before. When one curve lies

entirely outside the other, the portion of the latter which is swept at all will be swept both positively and negatively.

Hence if, as in the planimeter, one end A of AB is constrained to move along a line (whether circular or straight) without tracing out any area, the area of the curve traced out by the other end B will be equal to the resultant area swept out by the line AB, and hence will be bcn if AB return to its starting place without making a complete revolution. But if the fixed point of the planimeter lies inside the area described by the tracer B, the bar AB must make a complete revolution. Hence, the area described by B

$$= bcn + 2\pi bk + \text{circle described by } A$$

$$= bcn + 2\pi bk + \pi a^2$$

$$= bc\left[n + \frac{2\pi bk + \pi a^2}{bc}\right]$$

The second term in the brackets is a constant, and is engraved on the bar. This number is then to be looked upon as a correction to n when the planimeter makes a complete revolution. In certain cases n may be negative.

Let the distance of the wheel from the joint A be called d. Then (Fig. 1) $k = \frac{b}{2} - d$. Hence area described by B

$$= bcn + 2\pi b\left(\frac{b}{2} - d\right) + \pi a^2$$

$$= bcn + \pi\,(a^2 + b^2 - 2bd.)$$

Place the planimeter so that the perpendicular from the fixed point to the sweeping bar passes through the wheel. Then the square of the distance from the fixed point to

the tracer $B = a^2 + b^2 - 2bd$. $\therefore$ the area of the curve which B traces $= bcn$ + area of circle described by the tracer when the planimeter is in the position just indicated. This circle is called the datum circle. (In describing this circle the wheel would slide and not revolve, and hence n would be 0.)

Let it be required to find b so that the area may be found in square centimetres by multiplying n by 100.

We have $$bcn = \text{area} = 100\,n$$
$$\therefore bc = 100$$
$$\text{or, } b = \frac{100}{c} \text{ centimetres,}$$

where c is the circumference of the wheel in cm.

Similarly $$b = \frac{10}{c} \text{ inches}$$

if c = the circumference in inches, and the area is to be found in square inches by multiplying n by 10.

In general, if the area $= n$, bc is the unit area.

APPENDIX II.

THEORY OF THE MECHANICAL INTEGRATOR.

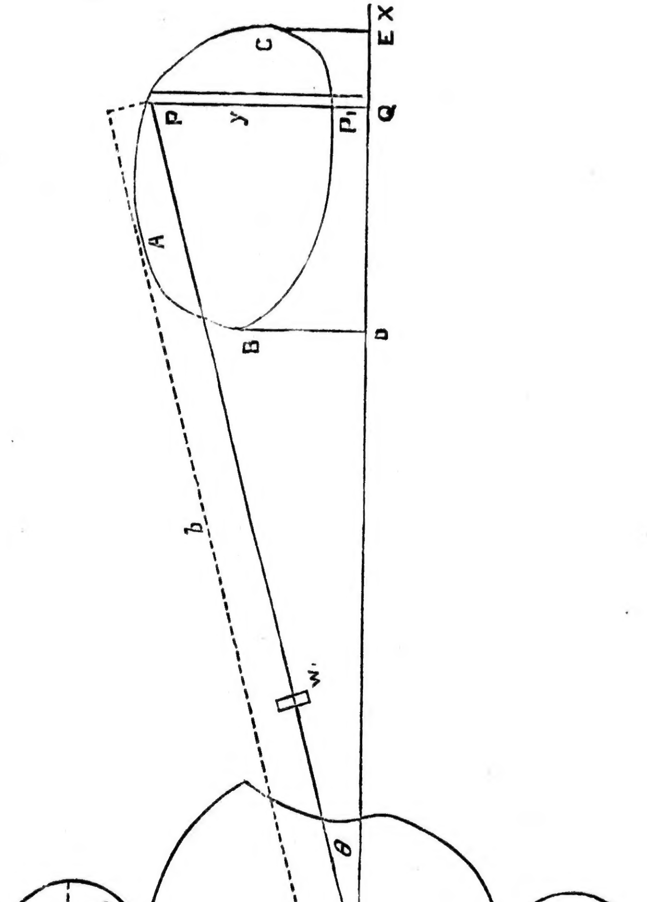
As we proceed from B to C by way of A (see figure), x changes from OD to OE and $\int ydx$ is the area $DBACE$; but, if we proceed from C to B by way of P_1, each element of area such as ydx is negative since dx is negative, and hence $\int ydx$ is the area $CEDB$, but is negative. Hence, if we sum the elements such as ydx in the order of proceeding clockwise round the curve, the result $= DBACE - BDEC = BAPC$, the area of the curve. Let A = this area, M = sum of the moments of the elements of the area with respect to OX, I = moment of inertia of the area, also with respect to OX. Then

$$A = \int ydx,$$

$$M = \int ydx \cdot \frac{y}{2} = \frac{1}{2}\int y^2\, dx,$$

$$I = \int ydx \cdot \frac{y^2}{3} = \frac{1}{3}\int y^3\, dx.$$

In the integrator one end of a sweeping bar traces a closed curve while the other end is constrained to describe a straight line OX. This part of the instrument is therefore a planimeter, and the area of the curve $= bc_1n_1$, where b is the length of the sweeping bar, c_1 the circumference of the wheel W_1, which the bar carries, and n_1 the change of reading of this wheel when the circuit of the curve has been made. The end of the bar which describes the


X
E
C
Q
P
y
P_1
A
B
D
b
W_1
θ
3θ
W_3
a
3a
2a
a
W_2
Y
O

straight line OX is also the centre of two arcs of radii $2a$ and $3a$ which turn circles each of radius a, the circles carrying wheels W_2 and W_3. When the axis of W_1 makes an angle θ with OX the axes of the other wheels make angles $\frac{\pi}{2} - 2\theta$ and 3θ, respectively, with the same line.

For y substitute $b \sin \theta$. Then

$$A = b \int \sin \theta \, dx,$$

$$M = \frac{1}{2} b^2 \int \sin^2 \theta \, dx,$$

$$I = \frac{1}{3} b^3 \int \sin^3 \theta \, dx.$$

Now $2 \sin^2 \theta = 1 - \cos 2\theta = 1 - \sin\left(\frac{\pi}{2} - 2\theta\right)$,

and $\sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta$,

or $\sin^3 \theta = \frac{3}{4} \sin \theta - \frac{1}{4} \sin 3\theta$.

$$\therefore M = \frac{1}{4} b^2 \int \left[1 - \sin\left(\frac{\pi}{2} - 2\theta\right)\right] dx,$$

$$= \frac{1}{4} b^2 \left[\int dx - \int \sin\left(\frac{\pi}{2} - 2\theta\right) dx\right]$$

$$= -\frac{1}{4} b^2 \int \sin\left(\frac{\pi}{2} - 2\theta\right) dx$$

since $\int dx = 0$ for the complete circuit of the curve.

Also $$I = \frac{1}{3} b^3 \int \left(\frac{3}{4} \sin \theta - \frac{1}{4} \sin 3\theta\right) dx$$

$$= \frac{1}{4} b^3 \int \sin \theta \, dx - \frac{1}{12} \int \sin 3\theta \, dx.$$

But by the theory of the planimeter the area $A = bc_1 n_1$.

$$\therefore \int \sin \theta \, dx = c_1 n_1,$$

i.e., when the axis of a wheel makes an angle θ with OX we have $\int \sin \theta \, dx = c_1 n_1$. But the axes of the other wheels make angles $\frac{\pi}{2} - 2\theta$ and 3θ with OX.

$$\therefore \int \sin\left(\frac{\pi}{2} - 2\theta\right) dx = c_2 n_2,$$

$$\text{and } \int \sin 3\theta \, dx = c_3 n_3.$$

$$\therefore A = bc_1 n_1,$$

$$M = \frac{1}{4} b^2 c_2 n_2,$$

$$I = \frac{1}{4} b^3 c_1 n_1 - \frac{1}{12} b^3 c_3 n_3.$$

The maker has constructed the machine so that $b = 2$ decimetres, $c_1 = \frac{1}{2}$ dec., $c = c_3 = \frac{3}{5}$ dec.

$$\therefore A = n_1,$$

$$M = \frac{3}{5} n_2,$$

$$I = n_1 - \frac{2}{5} n_3.$$

e

s

z